Indexing inexact proximity search with distance regression in pivot space

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The problem

Our task: Speed up proximity search in cases where:

- Distance calculation is expensive.
- Distance-based indexing is needed, because the contents of the data objects cannot be used in the index.
- Some search inexactness is acceptable, meaning we are allowed to trade som search accuracy in return for reduced search computation cost.

Our contribution: A new indexing scheme that in some cases provides better computation / accuracy trade-offs than the competition. It also has some draw-backs.

Related work

E. Chávez, K. Figueroa, and G. Navarro.
Effective proximity retrieval by ordering permutations.
IEEE Transactions on Pattern Analysis and Machine Intelligence, 30(9):1647–1658, 2008.

Takeaway:

- How to perform inexact search by ordering the database according to *promise value* function.
- A specific promise value function, which we use as a baseline in our experiments.
- Experimental setup.

Pivot space

Pivot set:

$$\mathbb{P} = (p_1, p_2, \ldots, p_m)$$

Mapping object o to pivot space.

$$\Phi(o) = (d(o, p_1), d(o, p_2), \ldots, d(o, p_m))$$

The baseline: Permutation based promise values

The promise value for indexed object u with respect to query q is the correlation (rank correlation coefficient, Spearman's ρ) between the ordering permutation of $\Phi(u)$ and that of $\Phi(q)$.

What makes a good promise value function?

Two ideas for promise value functions

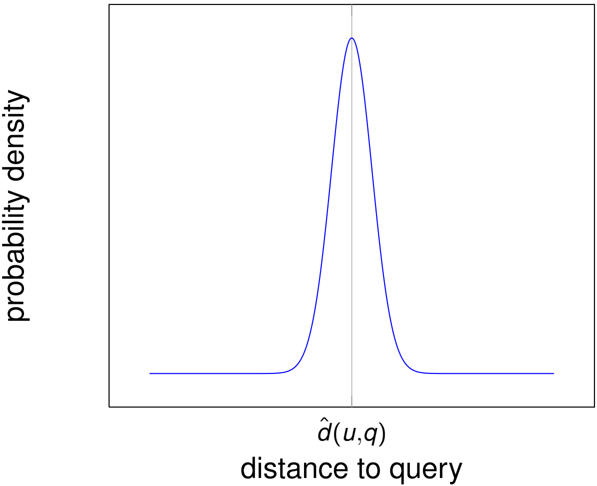
Distance estimate

$$\hat{d}(u, q)$$

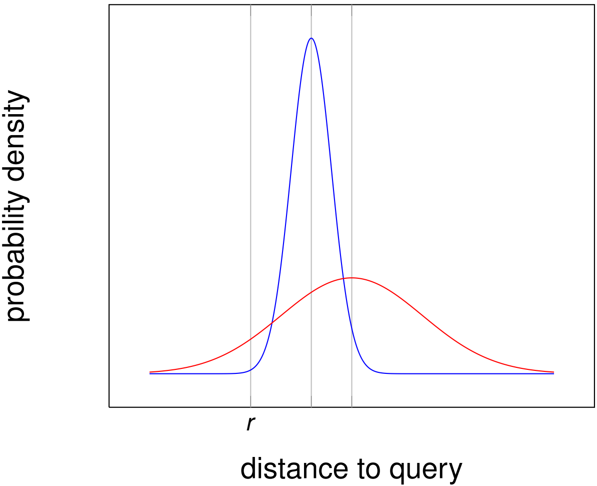
Probability of inclusion

$$\Pr(d(u, q) \leq r)$$

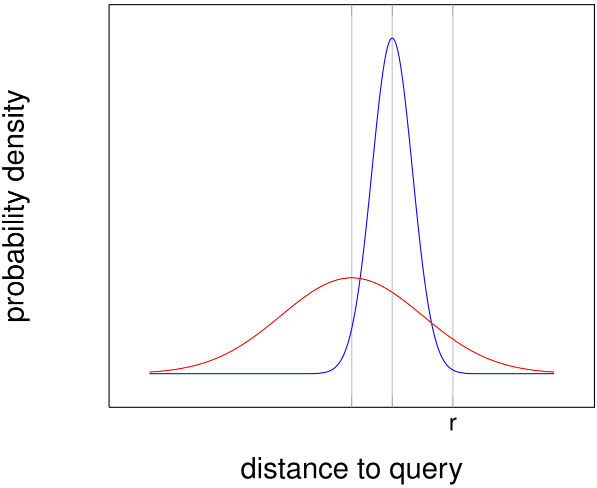
Uncertainty in distance estimates



Should red or blue object be visited first?



Should red or blue object be visited first?



Linear model of distance for indexed object u

$$d(u,q) = \beta_{\langle u,0\rangle} + \sum_{i=1}^{m} \beta_{\langle u,i\rangle} d(q,p_i) + \epsilon_u \,,$$

Regression-based index (one model per object!)

With n objects to index and m pivots, an $n \times (m+1)$ matrix:

$$\begin{pmatrix} \hat{\beta}_{\langle u_1,0\rangle}, \hat{\beta}_{\langle u_1,1\rangle}, \ldots, \hat{\beta}_{\langle u_1,m\rangle} \\ \hat{\beta}_{\langle u_2,0\rangle}, \hat{\beta}_{\langle u_2,1\rangle}, \ldots, \hat{\beta}_{\langle u_2,m\rangle} \\ \ldots \\ \hat{\beta}_{\langle u_n,0\rangle}, \hat{\beta}_{\langle u_n,1\rangle}, \ldots, \hat{\beta}_{\langle u_n,m\rangle} \end{pmatrix}$$

- The coefficients can be discretized to save space.
- Plus $2n+2$ additional values if we want probabilities.

Building the index

1. Select n' *training queries* from the objects to be indexed.
2. For each training query q', calculate $\Phi(q')$.
3. For each object to be indexed u:
   3.1 For each training query q', calculate $d(u, q')$.
   3.2 Solve the least squares linear regression problem to find the $m + 1$ coefficients β_u.
   3.3 Store the coefficients in the index.
   3.4 If we want probabilities, also store $\hat{\sigma}_u$, the estimated standard deviation of ϵ_u:

$$\hat{\sigma}_u = \sqrt{\frac{\sum_{i=1}^{n'} (d(u, q'_i) - \hat{d}(u, q'_i))^2}{n' - m - 1}}$$

4. If we want probabilities for the k-NN queries, also store the estimated search radius for each k.

(Detail glossed over in this presentation: we exclude u from the training queries used to fit its own model.)

Distance estimates as promise values

$$\hat{d}(u,q) = \hat{\beta}_{\langle u,0\rangle} + \sum_{i=1}^{m} \hat{\beta}_{\langle u,i\rangle} d(q,p_i)$$

Probability-based promise values

$$\frac{r - \hat{d}(u, q)}{\hat{\sigma}_u}$$

- Depends on a lot of assumptions.
- We also ignore the consequence of excluding *u* from its own training queries.

Storage costs

With n objects to index and m pivots,

- For distance estimates: $n(m+1)$ coefficients. (Can be discretized at the cost of some accuracy.)
  - For probabilities: $2n+2$ additional values.
- Permutation-based index: $nm\lceil\log_2(m)\rceil$ bits in total.

Index building costs

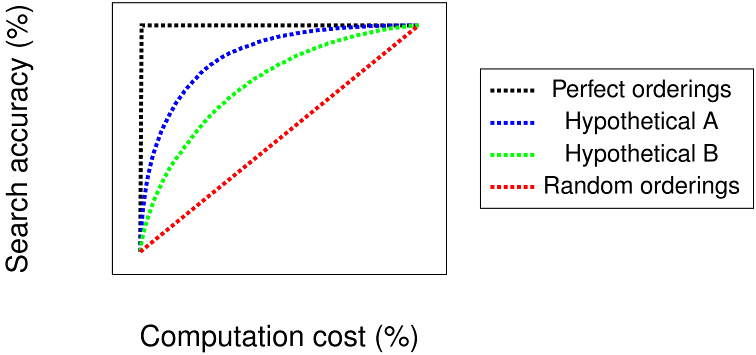
With n objects to index, m pivots and n' training queries,

- Regression-based scheme: $n'(n+m)$ distance calculations plus the solution of n linear regression problems.
- Permutation-based scheme: nm distance calculations, plus some sorting.

Experimental setup

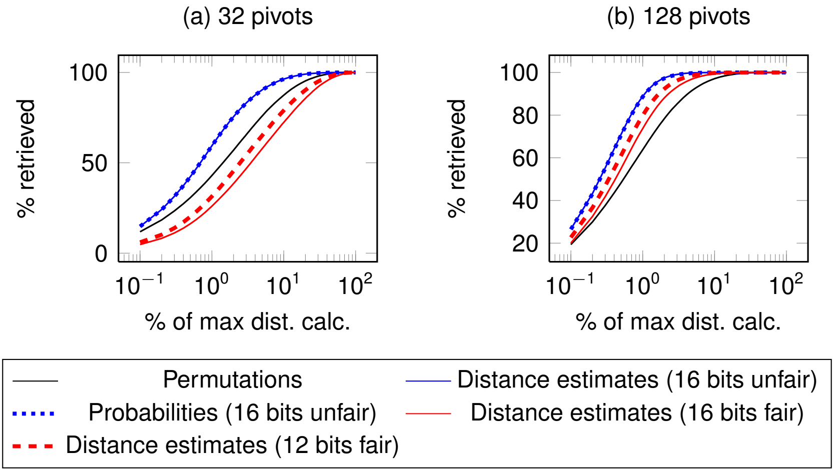
- We borrowed the experimental setup from Chávez, Figueroa & Navarro's evaluation of the permutation-based scheme.
- Pivots selected randomly.
- Also evaluated versions with pivot set reduced to make storage cost equal to permutation-based index.
- Both synthetic and real-world data sets, but results on real-world data may have more validity.

Evaluating promise value functions: computation / accuracy trade-offs

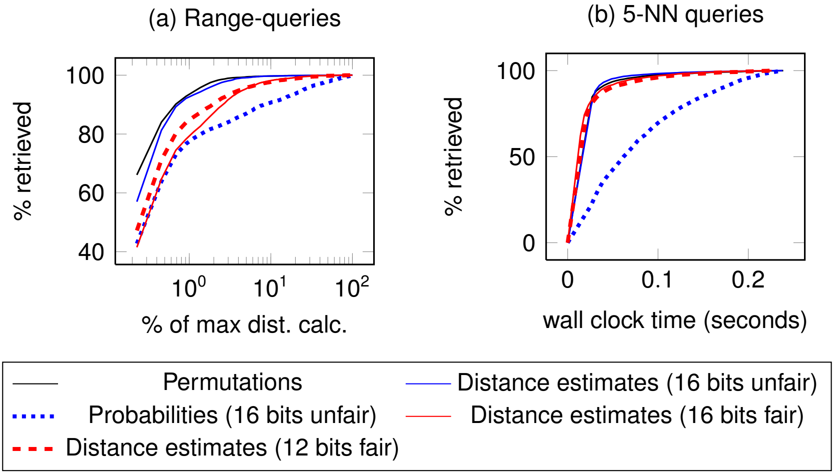


(Average over many queries.)

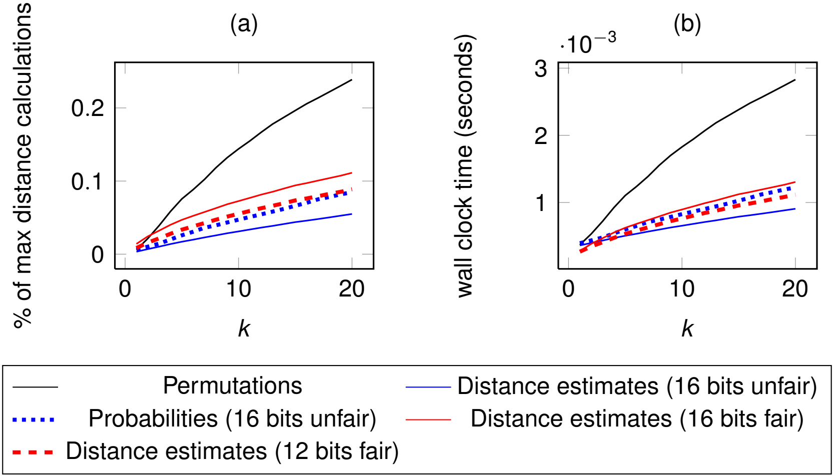
Results on normalized edit distance (NED)



Results on documents (TREC)



Results on face images (FERET)



Why were the probability-based promise values sometimes worse, and never better, than the distance estimates?

Conclusion

Regression-based scheme show some promise, but:

- Takes a lot of time to build the index.
- Vulnerable to deviation from assumptions.