# Metric Index: An Efficient and Scalable Solution for Similarity Search

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# Outline of the Talk

Efficient and scalable metric-based index is still an issue

15 years of research:

- principles of space partitioning, search space pruning, filtering
- fundamental memory-based structures
- advanced index and search solutions
- approximate similarity search
- distributed index structures

# Metric Index

Intentions of M-Index:

- synergically employ practically all known metric principles of space pruning and filtering
- fixed building costs (static set of reference points)
- use well-established efficient structures to factually store data
  - indexing based on mapping to real domain (use $B^+$-tree)
- efficient precise and approximation similarity search
- straightforward way to distribute the structure

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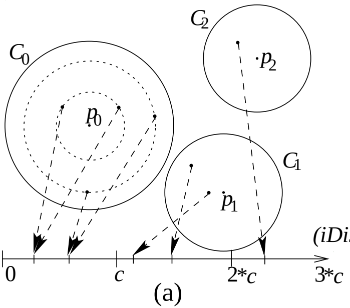
Notation and assumptions:

- Metric space $\mathcal{M}$ is a pair $\mathcal{M} = (\mathcal{D}, d)$, where $\mathcal{D}$ is a set and $d$ is a total function $d : \mathcal{D} \times \mathcal{D} \longrightarrow [0, 1)$ satisfying standard metric space conditions.

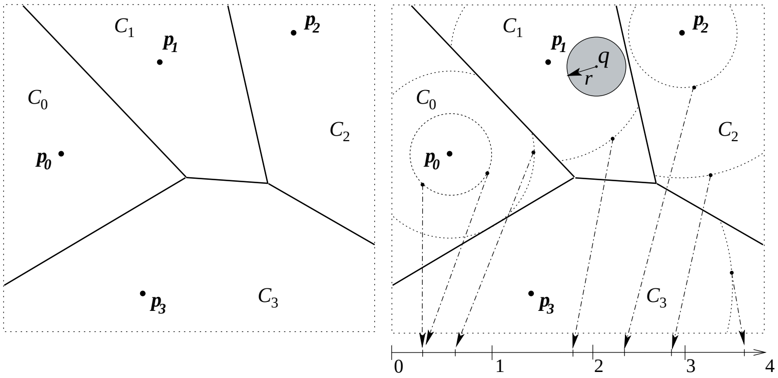
# Preliminaries: iDistance




$$iDist(o) = d(p_i, o) + i \cdot c$$

- indexing technique for vector spaces
- application of object-pivot distance constraint

# M-Index Level One




- index based purely on metric principles
- Voronoi partitioning
- double-pivot distance constraint for search-space pruning

# Multi-level M-Index (1)

Having a fixed set of $n$ pivots $\{p_0, p_1, \ldots, p_{n-1}\}$ and an object $o \in \mathcal{D}$, let

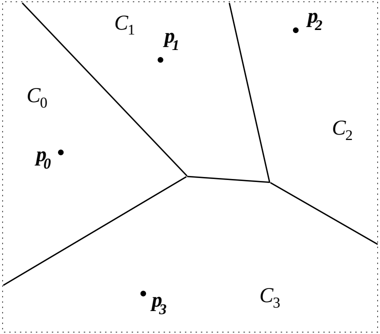
$$(\cdot)_o : \{0, 1, \ldots, n-1\} \longrightarrow \{0, 1, \ldots, n-1\}$$

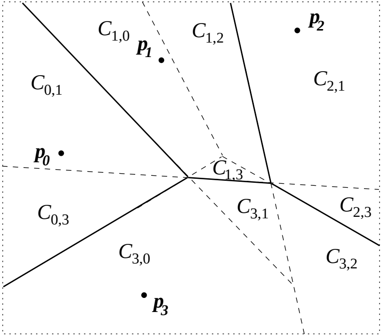
be a permutation of indexes such that

$$d(p_{(0)_o}, o) \leq d(p_{(1)_o}, o) \leq \cdots \leq d(p_{(n-1)_o}, o).$$

$l$-level M-Index uses $l$-prefix of the pivot permutation, $1 \leq l \leq n$.

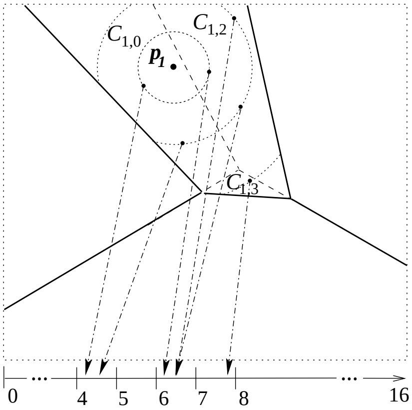
# Multi-level M-Index (2)




- in *l*-level M-Index, $\forall o \in \mathcal{D} : o \in C_{(0)_o,\ldots,(l-1)_o}$
- repetitive application of double-pivot distance constraint

# Multi-level M-Index Mapping




$$key_l(o) =$$
$$= d(p_{(0)_o}, o) + \sum_{i=0}^{l-1} (i)_o \cdot n^{(l-1-i)}$$

integral part of the key

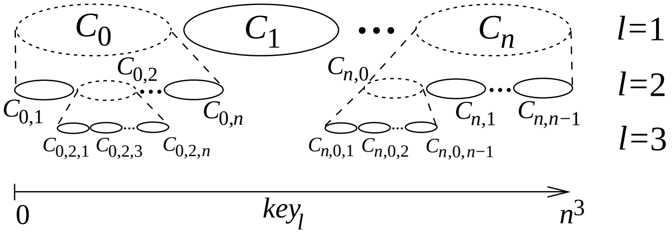
- identification of the cluster

fractional part of the key

- position within the cluster

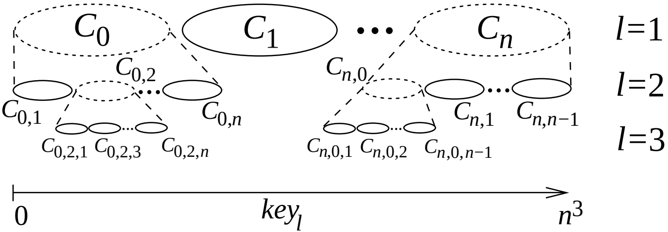
size of the $key_l$ domain: $n^l$

# M-Index with Dynamic Level




- establish a maximum level $1 \leq l_{max} \leq n$
- slightly modify the $key_l$ formula
  - analogous to *extensible hashing* + object-pivot distance

# M-Index with Dynamic Level




- establish a maximum level $1 \leq l_{max} \leq n$
- slightly modify the $key_l$ formula
  - analogous to *extensible hashing* + object-pivot distance
- physically store the data according to the key
  - in a $B^+$-tree or similar structure

# M-Index Search Principles

Search space pruning principles for $R(q, r)$ query

- double-pivot distance constraint
  - skip accessing of cluster $C_i$ if

$$d(p_i, q) - d(p_{(0)_q}, q) > 2 \cdot r$$

  - due to recursive Voronoi partitioning
  - apply $l$-times for $C_{i_0,\ldots,i_{l-1}}$

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  - due to recursive Voronoi partitioning
  - apply $l$-times for $C_{i_0,\dots,i_{l-1}}$

- range-pivot distance constraint
  - for leaf-level clusters $C_{p,*}$ store min. and max. distance

$$r_{\max} = \max\{d(p, o) | o \in C_{p,*}\}$$

  - skip accessing of cluster $C_{p,*}$ if

$$d(p, q) + r < r_{\min} \qquad \text{or} \qquad d(p, q) - r > r_{\max}$$

# M-Index Search Principles (cont.)

- object-pivot distance constraint
  - the M-Index key contains the object-pivot distance
  - identify interval of keys in cluster $C_{i_0,\ldots,i_{l-1}}$

$$[d(p_{i_0}, q) - r, d(p_{i_0}, q) + r]$$

# M-Index Search Principles (cont.)

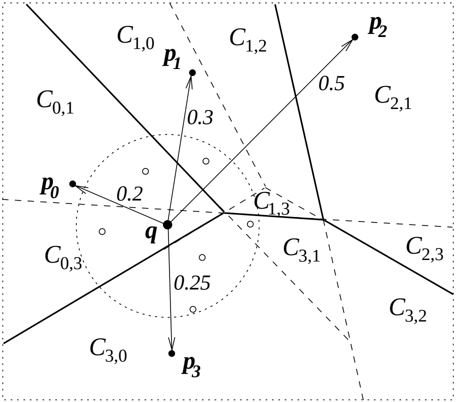
- object-pivot distance constraint
  - the M-Index key contains the object-pivot distance
  - identify interval of keys in cluster $C_{i_0,\ldots,i_{l-1}}$

$$[d(p_{i_0}, q) - r, d(p_{i_0}, q) + r]$$

- pivot filtering
  - store distances $d(p_0, o), \ldots, d(p_{n-1}, o)$ together with object $o$
  - skip computation of $d(q, o)$ at query time if

$$\max_{i \in \{0,\ldots,n-1\}} |d(p_i, q) - d(p_i, o)| > r.$$

# Approximate Strategy for M-Index




Determine the order in which to visit individual clusters

- priority queue of clusters
- heuristic which analyzes distances $d(p_0, q), d(p_1, q), \ldots, d(p_{n-1}, q)$
- each cluster is assigned a penalty
  - *distance of the cluster* from $q$

$$penalty(C_{i_0,\ldots,i_{l-1}}) = \sum_{j=0}^{l-1} \max\left\{d(p_{i_j}, q) - d(p_{(j)_q}, q), 0\right\}$$

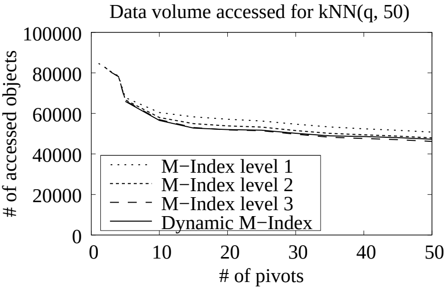
# Experimental Evaluation

- 200,000 objects from CoPhIR dataset
  - combination of five MPEG-7 descriptors
  - altogether 280 dimensions, intrinsic dimensionality: 13
- measure I/O costs (page reads and objects accessed), computational costs, response times
- compare with iDistance, PM-tree (same implementation platform)

Distance computations performed during index construction ($n = 20$)

| **dataset size** | 20,000 | 80,000 | 140,000 | 200,000 |
|---|---|---|---|---|
| **M-Index** | 400,000 | 1,600,000 | 2,800,000 | 4,000,000 |
| **PM-Tree** | 1,205,538 | 6,299,207 | 11,627,729 | 16,897,996 |

# Precise Strategy: Number of objects accessed




- dataset size: 100,000
- dynamic M-Index: $l_{max} = 5$

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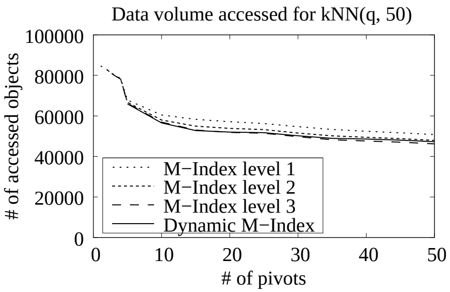
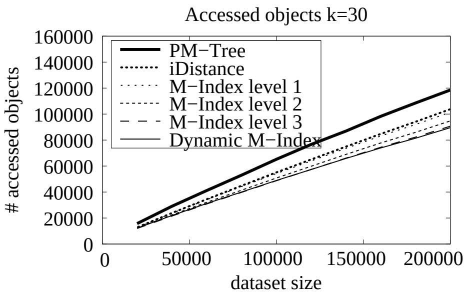
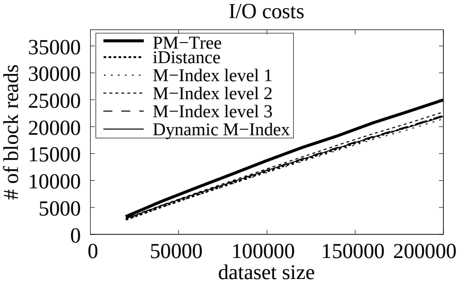



- dataset size: 100,000
- dynamic M-Index: $l_{max} = 5$

- 20 pivots

# Precise Strategy: I/O costs, response times




- higher fragmentation with more M-Index levels

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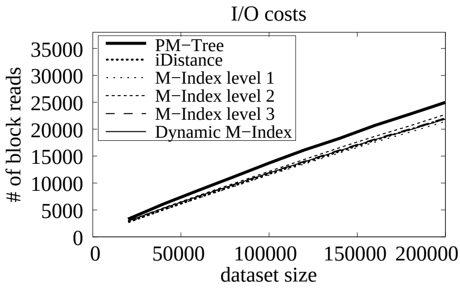
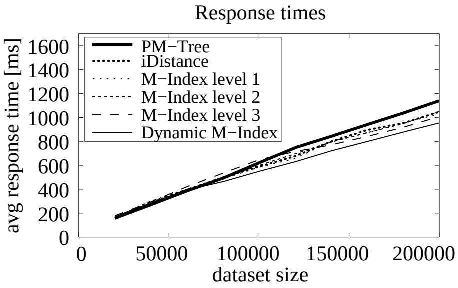
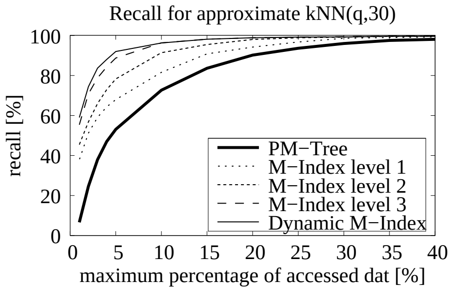



- higher fragmentation with more M-Index levels
- lower computational costs for more M-Index levels

# Approximate Strategy




- dataset of 100,000 objects

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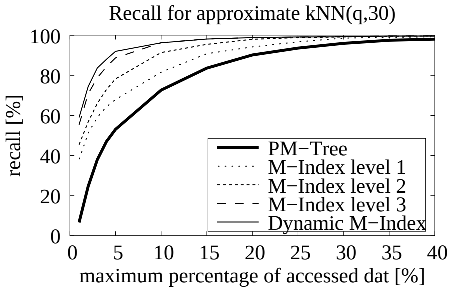


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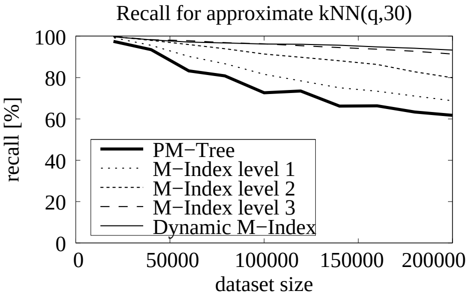
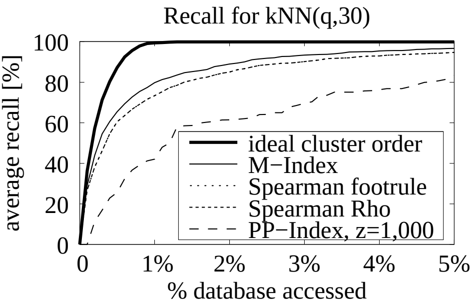



- algorithm accesses 10,000 objects

# Comparison with Purely Approximate Approaches




- dataset size: 1,000,000
- 32 pivots ($n = 32$)
- dynamic M-Index with $l_{\text{max}} = 6$

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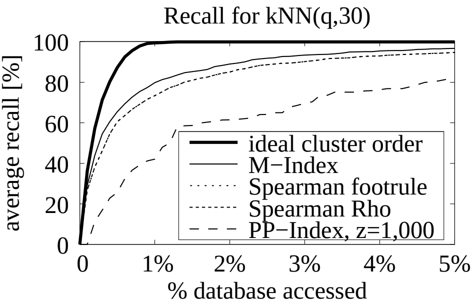


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- Spearman Footrule modified to use permutation prefixes
  - Induced Footrule Distance [*Amato, Savino: Approximate Similarity Search in Metric Spaces using Inverted Files*]
- Spearman Rho used in [*Chavez, Figueroa, Navarro: Effective Proximity Retrieval by Ordering Permutations*]
- PP-Index has a similar structure as M-Index
  - access subtrees with at least $z = 1{,}000$ objects